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C3T (Electricity and Magnetism) , Topic :- Network Theorems

❖ Introduction :

We shall discuss here some useful theorems which facilitate the analysis of networks. Specifically, we shall consider the superposition theorem, Thevenin's and Norton's theorems, and the maximum power transfer theorem with reference to AC circuits.

(1) Superposition Theorem

Statement: When a network containing linear, bilateral, passive circuit elements is excited simultaneously by a number of independent energy sources, the total response is the sum of the individual responses with each independent source acting separately, the other independent sources being deactivated and replaced by their internal impedances.

(2) Thevenin's Theorem

Statement: In a circuit containing linear, bilateral, passive circuit elements and energy sources, the current in a load impedance is the same as that supplied by a single voltage source of open-circuit voltage V_g and terminal impedance Z_g. V_g is the open-circuit voltage at the load terminals and Z_g is the impedance of the circuit looking back these terminals with all the energy sources deactivated and replaced by their internal impedances.

- **Proof :** Consider a circuit driven by a voltage sources of generated voltage V_B and internal impedance z_i. The circuit is connected to a load impedance z_L. If , instead of a voltage source, the circuit is excited by a current source ,this current source can be replaced by an equivalent voltage source. Furthermore, with respect to the driving point and the load terminals, the circuit can be reduced to a T or a π structure. Hence a T or a π circuit can be used to establish Thevenin's theorem. We consider a T circuit here shown in Fig. 1.

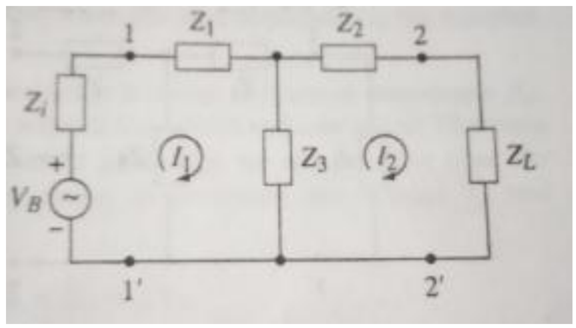

Fig.1 Circuit to prove Thevenin's Theorem

The reference polarities of V_B and the reference directions of the chosen loop currents I_1 and I_2 are shown in Fig. 1. The loop equations are as follows:

$$(Z_1 + Z_3 + Z_i)I_1 - Z_3 I_2 = V_B \quad \ldots.. (1)$$
$$(Z_2 + Z_3 + Z_L) - Z_3 I_1 = 0 \quad \ldots\ldots (2)$$

Solving eqs. (1) and (2) by Cramer's rule we have for the load current phasor

$$I_2 = \frac{\begin{vmatrix} Z_1+Z_3+Z_i & V_B \\ -Z_3 & 0 \end{vmatrix}}{\begin{vmatrix} Z_1+Z_3+Z_i & -Z_3 \\ -Z_3 & Z_2+Z_3+Z_L \end{vmatrix}} = \frac{Z_3 V_B}{(Z_1+Z_3+Z_i)(Z_2+Z_3+Z_L) - Z_3^2}$$

$$= \frac{Z_3 V_B}{(Z_1+Z_3+Z_i)(Z_2+Z_L) + Z_3(Z_1+Z_i)} \qquad \ldots. (3)$$

We shall now remove the load impedance z_L and find the open-circuit voltage phasor V_g at the load terminals 2,2'. The network of Fig.1 now reduces to that of Fig.2. The current through Z_2 being zero, the voltage V_g is the same as that across z_3. The current through z_3 is

$$I = \frac{V_B}{Z_1+Z_3+Z_i} \text{ , giving}$$

$$V_g = IZ_3 = \frac{V_B Z_3}{Z_1+Z_3+Z_i} \qquad \text{........ (4)}$$

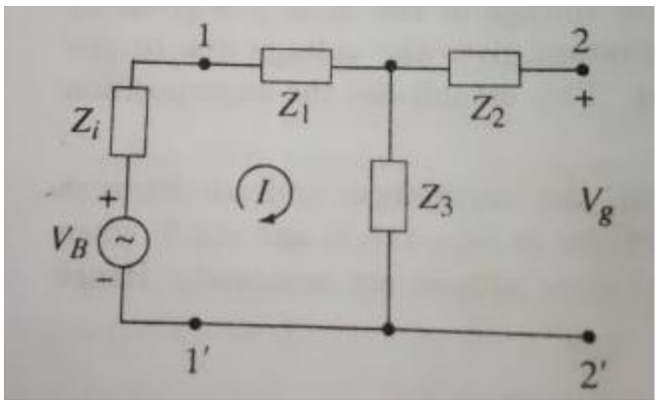

Fig.2 Circuit to find V_g

To obtain Z_g, the voltage source is deactivated and replaced by its internal impedance Z_i. Then Z_g is the impedance of the circuit of Fig.3 looking back from the load terminals 2,2'. Thus

$$Z_g = Z_2 + \frac{Z_3(Z_1+Z_i)}{(Z_1+Z_3+Z_i)} \qquad \text{......... (5)}$$

Now we connect the load impedance Z_L across a voltage source of open-circuit voltage V_g and internal impedance Z_g, as in Fig.4.The current through Z_L is

$$\boxed{I_L = \frac{V_g}{Z_g+Z_L} = \frac{V_B Z_3}{(Z_1+Z_3+Z_i)(Z_2+Z_L)+Z_3(Z_1+Z_i)}} \qquad \text{.......(6)}$$

Substituting for V_g and Z_g from equn. (4) and (5) , respectively . Equations (3) and (6) agree, thus establishing Thevenin's theorem. The voltage source of Fig.(4) is known as the equivalent ***Thevenin source,*** V_g is called the ***Thevenin voltage*** and Z_g, the ***Thevenin impedance.***

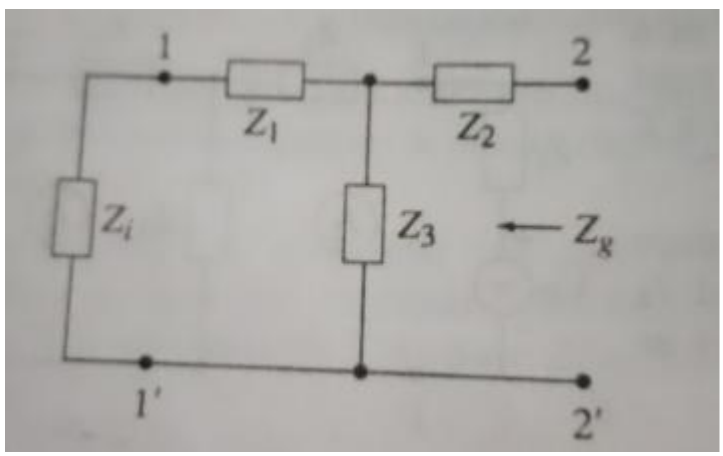

Fig.3 Circuit to Find Z_g

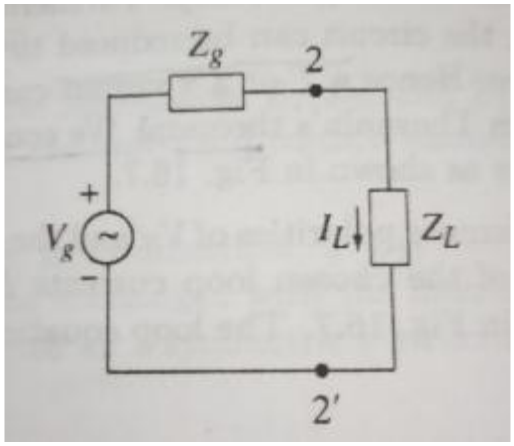

Fig.4 Equivalent Thevenin circuit

Observe that the current I_g flowing in 2,2' in Fig.1 when the terminals 2,2' are short circuited is the same as that at the load terminals in Fig.4 with $z_L = 0$. So, $I_g = \frac{V_g}{Z_g}$, or $Z_g = \frac{V_g}{I_g}$. Hence Z_g is the open-circuit voltage divided by the short circuit current at the load terminals in the original circuit. This offers an *alternative method* of calculating the Thevenin impedance.

If the circuit elements are nonlinear and unilateral, the impedance values will be determined by the current through them. Hence the values of the impedances on the right –hand sides of Eqn. (3) and (6) will differ, and the two values of I_L will thus not agree. Clearly, Thevenin's theorem will not for such circuit elements.