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C3T (Electricity and Magnetism) , Topic :- Network Theorems(Part-II)

(3)Norton's Theorem

Statement: In a circuit containing linear, bilateral, passive circuit elements and energy sources, the current in a load impedance is the same as that supplied by a simple source of generated current $I_g$in parallel with an impedance z_g. I_g is the short-circuit current source at the load terminals and z_g is the impedance of the circuit looking back from these terminals with all the energy sources deactivated and replaced by their internal impedances.

- **Proof :** If we convert the Thevenin source of Fig.4 into an equivalent current source with respect to the the load terminals 2,2'.we obtain the circuit of Fig.5,which,in turn, is equivalent to the original circuit of Fig.(1) . In the circuit of the Fig.(5) $I_g = \frac{V_g}{Z_g}$ **,**which is the short-circuit current at the load terminals 2,2' Fig.(4) and hence Fig.(1). Thus load current I_L is supplied by the source current I_g shunted by $Z_g$which is the Thevenin impedance , i.e., the impedance looking back from the terminals 2,2' with all the energy sources deactivated and replaced by their internal impedance. This proves Norton's theorem.

This current source in Fig.(5) is called the equivalent ***Norton source.***

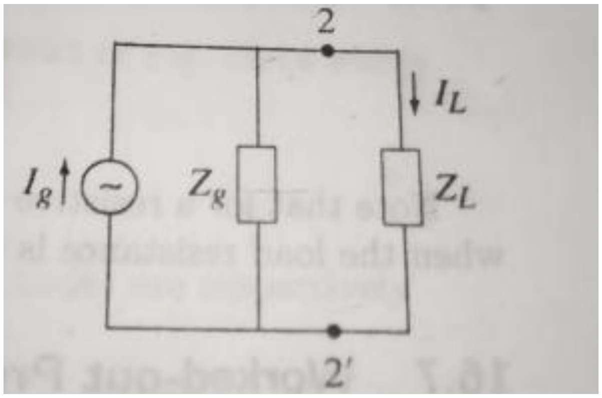

Fig.5

(4) Maximum Power Transfer Theorem

Statement: In a AC circuit, if the resistance and the reactance of the load can be varied independently, maximum power is delivered to the load impedance is the complex conjugate of the source impedance

- **Proof :** A load impedance may be directly connected to a source of internal impedance Z_g . It can also be connected to a circuit as in Fig.(1),whence the circuit reduces to the Thevenin equivalent form. Thus, without any loss of generality, we can consider that the load impedance Z_L is connected to an AC voltage source of generated rms voltage V_g and internal impedance Z_g. Let

$$\boldsymbol{Z_g = R_g + jX_g} \text{ and } \boldsymbol{Z_L = R_L + jX_L}$$

Where R_g is the resistance and X_g is the reactance component of Z_g, and R_L and X_L are the corresponding quantities for Z_L. The current phasor is

$$I_L = \frac{V_g}{Z_g + Z_L} = \frac{V_g}{(R_g + R_L) + j(X_g + X_L)}$$

The power delivered to the load is

$$P = |I_L|^2 R_L = \frac{|V_g|^2 R_L}{(R_g+R_L)^2+(X_g+X_L)^2} \quad \text{......(1)}$$

R_g and X_g are fixed, whereas R_L and X_L can be freely varied here. Obviously, P is a maximum if X_L is varied, when $X_L = -X_g$. Observe that this is the condition of series resonance. Under this condition, eqn.(1) gives

$$P = \frac{|V_g|^2 R_L}{(R_g+R_L)^2}$$

Now , P is maximized by varying R_L . Under the maximum power condition,

$$\frac{dp}{dR_L} = 0 \quad \text{giving } R_L = R_g$$

Thus for maximum power transfer to the load , we have $Z_L = R_L + jX_L = R_g - jX_g = Z_g^*$, where Z_g^* is the complex conjugate of the source impedance Z_g . This establishes the maximum power transfer theorem. The maximum power delivered to the load is

$$P_{max} = \frac{|V_g|^2}{4R_g} = \frac{|V_g|^2}{4R_L}$$

Note that for a resistive circuit, maximum power is transferred to a variable load resistance when the load resistance is simply equal to the reistance.

REVIEW QUESTIONS

1. Define (i) nonlinear, (ii) linear, (iii) unilateral (iv) bilateral (v) passive ,and (vi) active circuits elements.
2. (a) What is the internal impedance of (i) an ideal voltage source , and (ii) an ideal current source?

   (b) Explain how you can convert a voltage source into an equivalent current source, and vice versa.

3. State and prove the following theorems for AC circuits : (i) Superposition theorem, (ii) Thevenin's theorem, (iii) Norton's theorem, (iv) Maximum power transfer theorem.

PROBLEMS

1. In the circuit of the Figure, determine the source voltage V so that the power dissipated in the 4 ohm resistor is 1kW. [**Ans**. 122.9 V]

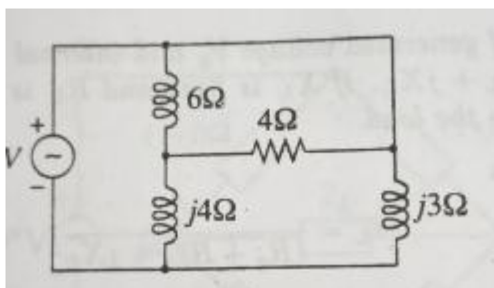

Figure for Problem 1

2. Find the Thevenin and Norton equivalent forms of the network of the Figure, between the terminals a, b . [**Ans.** $\mathbf{9.487\angle 18.43^0\ V, (1.5 - j1.5)\Omega\ ; 4.473\angle 63.43^0 A, (1.5 - j1.5)\Omega}$]

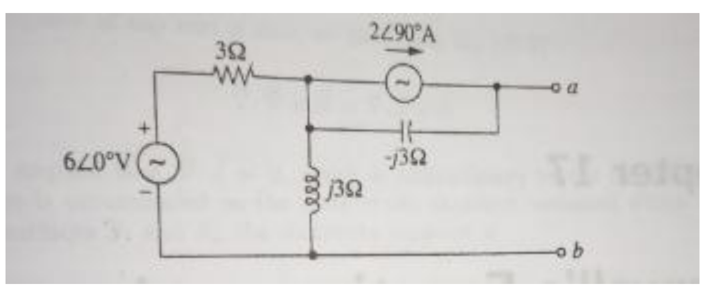

Figure for Problem 2