Differential Equation

Formation

(a) Make the differential Equⁿ of following curve

① $y = mx + c$; (m, c are constant)

$\Rightarrow \frac{dy}{dx} = m$ [diff: w.r.to x]

$\Rightarrow \frac{d^2y}{dx^2} = 0$ ["]

$y = \frac{a}{x} + b$, where a, b are constant

$$\Rightarrow \frac{d^2y}{dx^2} = 0 \quad \left[\quad " \quad \right]$$

(2) $y = \frac{a}{x} + b$, where a, b are constant.

Diff. w.r.t. x,

$$\frac{dy}{dx} = a\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow x^2 \frac{dy}{dx} = -a$$

Diff. w.r.to x;

$$x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = 0$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

③ $x^2+y^2=a^2$

Diff ___

$\Rightarrow 2x+2y\frac{dy}{dx}=0$

$\Rightarrow x+y\frac{dy}{dx}=0$ #

$\Rightarrow y\frac{dy}{dx}=-x$

$\Rightarrow \frac{dy}{dx}=-\frac{x}{y}$

$\Rightarrow \frac{dy}{dx}+\frac{x}{y}=0$

④ $xy = c^2$

Diff $\longrightarrow$

$$x\frac{dy}{dx} + y = 0$$ #

⑤ $x + y = b$

Diff. w.r.t. x $\longrightarrow$

$$1 + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -1$$ #

(8)

$$y = ae^{x} + be^{-x}$$

Diff. w.r.to x,

$$\frac{dy}{dx} = ae^{x} + (-b)e^{-x}$$

Diff. w.r.t. x,

$$\frac{d^2y}{dx^2} = ae^{x} + be^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = y \; \#$$

(8) $y = ae^{2x} + be^{-2x}$

Diff. w.r.t. x

$$\frac{dy}{dx} = 2ae^{2x} + (-2b)e^{-2x}$$

Diff. w.r.t. x,

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{-2x}$$

$$\frac{d^2y}{dx^2} = 4(ae^{2x} + be^{-2x})$$

$$\frac{d^2y}{dx^2} = 4y \quad \#$$

$$\Rightarrow \frac{d^2y}{dx^2} = y \quad \#$$

$$\frac{d^2y}{dx^2} = 4y \quad \#$$

(8) $y = e^x(a\cos x + b\sin x)$

Differentiating w.r.t x,

$$\frac{dy}{dx} = e^x(a\cos x + b\sin x) + e^x(a(-\sin x) + b\cos x)$$

$$\frac{dy}{dx} = y + e^x(-a\sin x + b\cos x) \quad \text{—(1)}$$

Diff. (1) w.r.t. to x,

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} = \left(\frac{dy}{dx} - y\right) - e^x(a\cos x + b\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} = -y - y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \quad \#$$

(8) $y = A\cos(\log x) + B\sin(\log x)$

Diff. w.r.to x,

$$\frac{dy}{dx} = -A\sin(\log x)\frac{1}{x} + B\cos(\log x)\frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$$

$\Rightarrow$ Diff. w.r.to x,

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -A\cos(\log x)\frac{1}{x} - B\sin(\log x)\frac{1}{x}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\left[A\cos(\log x) + B\sin(\log x)\right]$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$ #